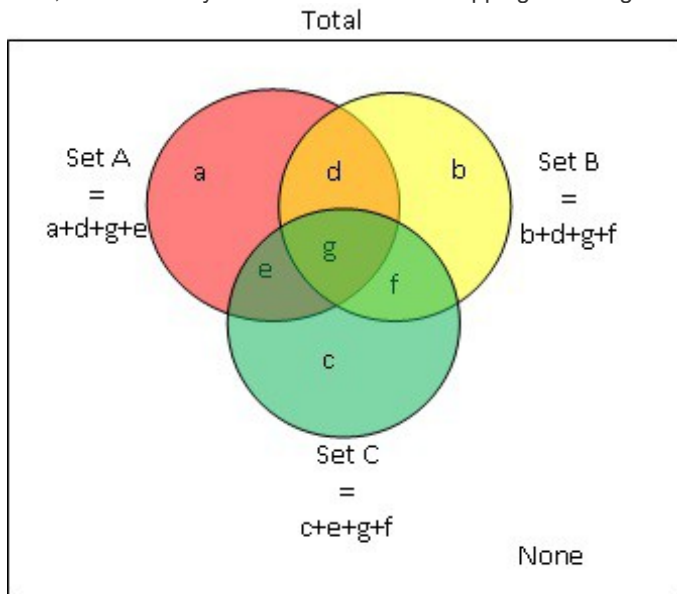


Today, let's take a look at how to use Venn diagrams to solve questions involving three overlapping sets. First, let me show you what the three overlapping sets diagram looks like.



Notice that the total comprises of the elements that do not fall in any of the three sets and the elements that are a part of at least one of the three sets.

The elements falling in the red, yellow or green region (region a, b or c) fall in only one set. The elements falling in region d, e or f fall in exactly two sets and the elements falling in region g fall in all three sets.

Now some quick questions to get a clear picture:

Question 1: Which regions represent the elements that belong to at least 2 sets?

Answer 1: $d + e + f + g$

Question 2: Which regions represent the elements that belong to at least 1 set?

Answer 2: $a + b + c + d + e + f + g = \text{Total} - \text{None}$

Question 3: Which regions represent the elements that belong to at most 2 sets?

Answer 3: $\text{None} + a + b + c + d + e + f = \text{Total} - g$

Hope there are no doubts up till now. Let's look at a question to see how to apply these concepts.

Question: Three table runners have a combined area of 200 square inches. By overlapping the runners to cover 80% of a table of area 175 square inches, the area that is covered by exactly two layers of runner is 24 square inches. What is the area of the table that is covered with three layers of runner?

- (A) 18 square inches
- (B) 20 square inches
- (C) 24 square inches
- (D) 28 square inches
- (E) 30 square inches

Solution: Let's first try to understand what exactly is given to us. The area of all the runners is equal to 200 square inches.

$$\text{Runner 1} + \text{Runner 2} + \text{Runner 3} = 200$$

In our diagram, this area is represented by

$$(a + d + g + e) + (b + d + g + f) + (c + e + g + f) = 200$$

(We need to find the value of g i.e. the area of the table that is covered with three layers of runner.)

Area of table covered is only 80% of 175 i.e. only 140 square inches. This means that if each section is counted only once, the total area covered is 140 square inches.

$$a + b + c + d + e + f + g = 140$$

So the overlapping regions are obtained by subtracting second equation from the first. We get $d + e + f + 2g = 60$

But $d + e + f$ (area with exactly two layers of runner) = 24

$$\text{So } 2g = 60 - 24 = 36$$

$g = 18$ square inches

Note that you don't need to make all these equations and can directly jump to $d + e + f + 2g = 60$. We wrote these equations down only for clarity. It is a matter of thinking vs solving. If we think more, we have to solve less. Let's see how. Combined area of runners is 200 square inches while area of table they cover is only 140 square inches. So what does the extra 60 square inches of runner do? It covers another runner!

Wherever there are two runners overlapping, one runner is not covering the table but just another runner. Wherever there are three runners overlapping, two runners are not covering the table but just the third runner at the bottom.

So can we say that $(d + e + f)$ represents the area where one runner is covering another runner and g is the area where two runners are covering another runner?

Put another way, can we say $d + e + f + 2g = 60$?

We know that $d + e + f = 24$ giving us $g = 18$ square inches

This entire 'thinking process' takes ten seconds once you are comfortable with it and your answer would be out in about 30 sec!